20

Nuno Andre/Shutterstock.com

TIME-SERIES ANALYSIS AND FORECASTING

CHAPTER OUTLINE

Housing Starts

DATA
Xm20-00

At the end of 2005, a major builder of residential houses in the northeastern United States wanted to predict the number of housing units that would be started in 2006. This information would be extremely useful in determining a variety of variables, including housing demand, availability of labor, and the price of building materials. To help develop an accurate forecasting model, an economist collected data on the number of housing starts (in thousands) for the previous 60 months (2001–2005). Forecast the number of housing starts for the 12 months of 2006. (*Source*: Standard & Poor's Industry Surveys.)*

iStockphoto.com/stocknroll

See page 853 for the answer.

*We have chosen not to update the housing starts data. In 2008, most of the world's economies were thrown into a recession precipitated by a housing crisis in the United States. As a result, statistical forecasting tools became wildly inaccurate. You can see for yourself how inaccurate by completing Exercise 20.52.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Introduction

Any variable that is measured over time in sequential order is called a **time series**. We introduced time series in Chapter 3 and demonstrated how we use a line chart to graphically display the data. Our objective in this chapter is to analyze time series in order to detect patterns that will enable us to forecast future values of the time series. There is an almost unlimited number of such applications in management and economics. Some examples follow.

1. Governments want to know future values of interest rates, unemployment rates, and percentage increases in the cost of living.
2. Housing industry economists must forecast mortgage interest rates, demand for housing, and the cost of building materials.
3. Many companies attempt to predict the demand for their products and their share of the market.
4. Universities and colleges often try to forecast the number of students who will be applying for acceptance at postsecondary-school institutions.

Forecasting is a common practice among managers and government decision makers. This chapter focuses on time-series forecasting, which uses historical time-series data to predict future values of variables such as sales or unemployment rates. This entire chapter is an application tool for both economists and managers in all functional areas of business because forecasting is such a vital factor in decision making in these areas.

For example, the starting point for aggregate production planning by operations managers is to forecast the demand for the company's products. These forecasts will make use of economists' forecasts of macroeconomic variables (such as gross domestic product, disposable income, and housing starts) as well as the marketing managers' internal forecasts of their customers' future needs. Not only are these sales forecasts critical to production planning but also they are the key to accurate pro forma (i.e., forecasted) financial statements, which are produced by the accounting and financial managers to assist in their planning for future financial needs such as borrowing. Likewise, the human resources department will find such forecasts of a company's growth prospects to be invaluable in their planning for future worker requirements.

There are many different forecasting techniques. Some are based on developing a model that attempts to analyze the relationship between a dependent variable and one or more independent variables. We presented some of these methods in the chapters on regression analysis (Chapters 16, 17, and 18). The forecasting methods to be discussed in this chapter are all based on time series, which we discuss in the next section. In Sections 20-2 and 20-3, we deal with methods for detecting and measuring which time-series components exist. After we uncover this information, we can develop forecasting tools. We will only scratch the surface of this topic. Our objective is to expose you to the concepts of forecasting and to introduce some of the simpler techniques. The level of this text precludes the investigation of more complicated methods.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

20-1 Time-Series Components

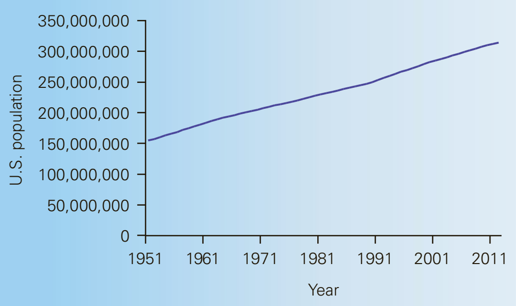
A time series can consist of four different components as described in the box.

Time-Series Components

1. Long-term trend
2. Cyclical variation
3. Seasonal variation
4. Random variation

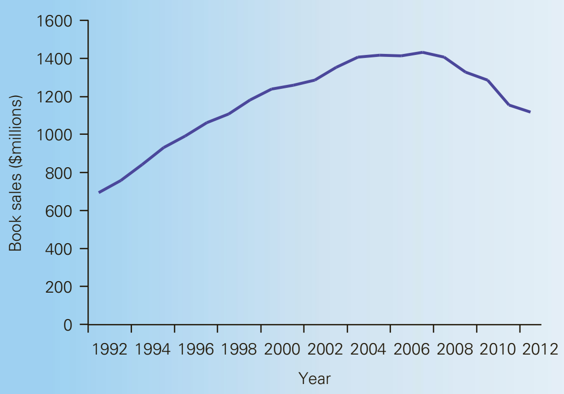
A **trend** (also known as a **secular trend**) is a long-term, relatively smooth pattern or direction exhibited by a series. Its duration is more than 1 year. For example, the population of the United States exhibited a trend of relatively steady growth from 157 million in 1952 to 314 million in 2012. (The data are stored in Ch20:\Fig20-01.) Figure 20.1 exhibits the line chart.

FIGURE **20.1** **U.S. Population (millions), 1952–2012**



The trend of a time series is not always linear. For example, Figure 20.2 describes U.S annual retail book sales in $millions. As you can see, sales increased from 1992 to 2008 and has decreased since then (data are stored in Ch20:\Fig20-02).

FIGURE **20.2** **U.S. Annual Retail Book Sales ($millions)**



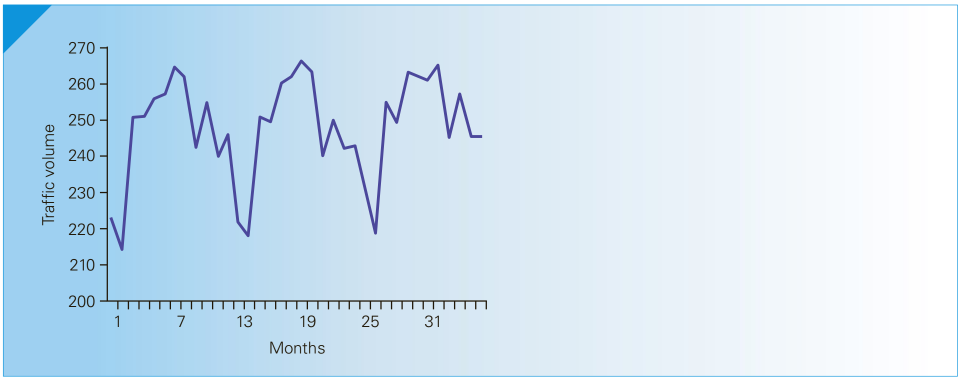
Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Cyclical variation is a wavelike pattern describing a long-term trend that is generally apparent over a number of years, resulting in a cyclical effect. By definition, it has duration of more than 1 year. Examples include business cycles that record periods of economic recession and inflation, long-term product-demand cycles, and cycles in monetary and financial sectors. However, cyclical patterns that are consistent and predictable are quite rare. For practical purposes, we will ignore this type of variation.

Seasonal variation refers to cycles that occur over short repetitive calendar periods and, by definition, have a duration of less than 1 year. The term *seasonal variation* may refer to the four traditional seasons or to systematic patterns that occur during a month, a week, or even one day. Demand for restaurants feature "seasonal" variation throughout the day.

An illustration of seasonal variation is provided in Figure 20.3, which graphs monthly U.S. traffic volume (in billions of miles and where period 1 is January). (Data are in Ch20:\Fig20-03.) It is obvious from the graph that Americans drive more during the summer months than during the winter months.

FIGURE **20.3** Traffic Volume (Billions of Miles)



Random variation is caused by irregular and unpredictable changes in a time series that are not caused by any other components. It tends to mask the existence of the other more predictable components. Because random variation exists in almost all time series, one of the objectives of this chapter is to introduce ways to reduce the random variation, which will enable statistics practitioners to describe and measure the other components. By doing so, we hope to be able to make accurate predictions of the time series.

20-2 Smoothing Techniques

If we can determine which components actually exist in a time series, we can develop better forecasts. Unfortunately, the existence of random variation often makes the task of identifying components difficult. One of the simplest ways to reduce random variation is to smooth the time series. In this section, we introduce two methods: *moving averages* and *exponential smoothing*.

20-2a Moving Averages

A **moving average** for a time period is the arithmetic mean of the values in that time period and those close to it. For example, to compute the three-period moving average for any time period, we would average the time-series values in that time period, the previous period, and the following period. We compute the three-period moving

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

averages for all time periods except the first and the last. To calculate the five-period moving average, we average the value in that time period, the values in the two preceding periods, and the values in the two following time periods. We can choose any number of periods with which to calculate the moving averages.

EXAMPLE 20.1

DATA
Xm20-01

Gasoline Sales, Part 1

As part of an effort to forecast future sales, an operator of five independent gas stations recorded the quarterly gasoline sales (in thousands of gallons) for the past 4 years. These data are shown below. Calculate the three-quarter and five-quarter moving averages. Draw graphs of the time series and the moving averages.

Time Period	Year	Quarter	Gasoline Sales (Thousands of Gallons)
1	1	1	39
2		2	37
3		3	61
4		4	58
5	2	1	18
6		2	56
7		3	82
8		4	27
9	3	1	41
10		2	69
11		3	49
12		4	66
13	4	1	54
14		2	42
15		3	90
16		4	66

SOLUTION:

COMPUTE

MANUALLY:

To compute the first three-quarter moving average, we group the gasoline sales in periods 1, 2, and 3, and then average them. Thus, the first moving average is

$$\frac{39+37+61}{3}=\frac{137}{3}=45.7$$

The second moving average is calculated by dropping the first period's sales (39), adding the fourth period's sales (58), and then computing the new average. Thus, the second moving average is

$$\frac{37+61+58}{3}=\frac{156}{3}=52.0$$

The process continues as shown in the following table. Similar calculations are made to produce the five-quarter moving averages (also shown in the table).

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Time Period	Gasoline Sales	Three-Quarter Moving Average	Five-Quarter Moving Average
1	39	–	–
2	37	45.7	–
3	61	52.0	42.6
4	58	45.7	46.0
5	18	44.0	55.0
6	56	52.0	48.2
7	82	55.0	44.8
8	27	50.0	55.0
9	41	45.7	53.6
10	69	53.0	50.4
11	49	61.3	55.8
12	66	56.3	56.0
13	54	54.0	60.2
14	42	62.0	63.6
15	90	66.0	–
16	66	–	–

Notice that we place the moving averages in the center of the group of values being averaged. It is for this reason that we prefer to use an odd number of periods in the moving averages. Later in this section, we discuss how to deal with an even number of periods.

Figure 20.4 displays the line chart for gasoline sales, and Figure 20.5 shows the three-period and five-period moving averages.

FIGURE **20.4** Quarterly Gasoline Sales

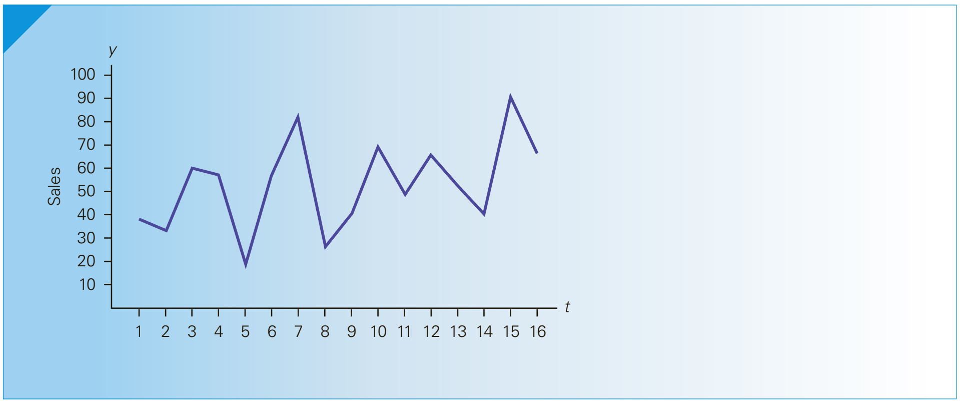
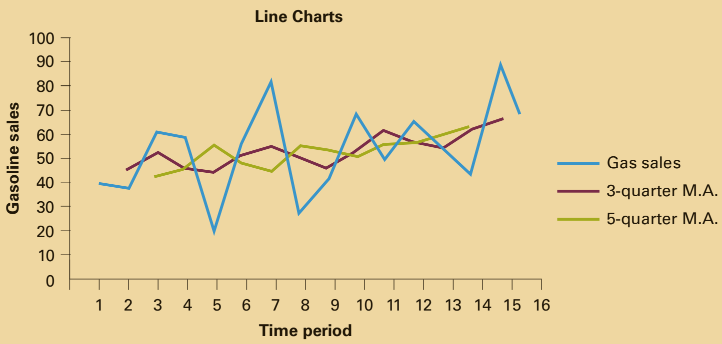


FIGURE **20.5** Quarterly Gasoline Sales and Three-Quarter and Five-Quarter Moving Averages

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXCEL Data Analysis

	A	B
1	Gas Sales	Moving Average
2	39	
3	37	45.7
4	61	52.0
5	58	45.7
6	18	44.0
7	56	52.0
8	82	55.0
9	27	50.0
10	41	45.7
11	69	53.0
12	49	61.3
13	66	56.3
14	54	54.0
15	42	62.0
16	90	66.0
17	66	



INSTRUCTIONS

1. Type or import the data into one column. (Open Xm20-01.)
2. Click **Data, Data Analysis,** and **Moving Average**.
3. Specify the **Input Range** (A1:A17). Specify the number of periods (3), and the **Output Range** (B1).
4. Delete the cells containing N/A.
5. To draw the line charts, follow the instructions on page 61.

INTERPRET

To see how the moving averages remove some of the random variation, examine Figures 20.4 and 20.5. Figure 20.4 depicts the quarterly gasoline sales. Discerning any of the time-series components is difficult because of the large amount of random variation. Now consider the three-quarter moving average in Figure 20.5. You should be able to detect the seasonal pattern that exhibits peaks in the third quarter of each year (periods 3, 7, 11, and 15) and valleys in the first quarter of the year (periods 5, 9, and 13). There is also a small but discernible long-term trend of increasing sales.

Notice also in Figure 20.5 that the five-quarter moving average produces more smoothing than the three-quarter moving average. In general, the longer the time period over which we average, the smoother the series becomes. Unfortunately, in this case we've smoothed too much—the seasonal pattern is no longer apparent in the five-quarter moving average. All we can see is the long-term trend. It is important to realize that our objective is to smooth the time series sufficiently to remove the random variation and to reveal the other components (trend, cycle, or season) present. With too little smoothing, the random variation disguises the real pattern. With too much smoothing, however, some or all of the other effects may be eliminated along with the random variation.

20-2b Centered Moving Averages

Using an even number of periods to calculate the moving averages presents a problem about where to place the moving averages in a graph or table. For example, suppose that we calculate the four-period moving average of the following time series:

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Period	Time Series
1	15
2	27
3	20
4	14
5	25
6	11

The first moving average is

$$\frac{15 + 27 + 20 + 14}{4} = 19.0$$

However, because this value represents time periods $1, 2, 3,$ and 4, we must place it between periods 2 and 3. The next moving average is

$$\frac{27 + 20 + 14 + 25}{4} = 21.5$$

and it must be placed between periods 3 and 4. The moving average that falls between periods 4 and 5 is

$$\frac{20 + 14 + 25 + 11}{4} = 17.5$$

There are several problems that result from placing the moving averages between time periods, including graphing difficulties. Centering the moving average corrects the problem. We do this by computing the two-period moving average of the four-period moving average. Thus, the centered moving average for period 3 is

$$\frac{19.0 + 21.5}{2} = 20.25$$

The centered moving average for period 4 is

$$\frac{21.5 + 17.5}{2} = 19.50$$

The following table summarizes these results.

Period	Time Series	Four-Period Moving Average	Four-Period Centered Moving Average
1	15	—	—
2	27	19.0	—
3	20	21.5	20.25
4	14	17.5	19.50
5	25	—	—
6	11		—

20-2c Exponential Smoothing

Two drawbacks are associated with the moving average method of smoothing time series. First, we do not have moving averages for the first and last sets of time periods. If the time series has few observations, the missing values can represent an important loss of information. Second, the moving average "forgets" most of the previous time-series values. For example, in the five-quarter moving average described in Example 20.1,

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

the average for quarter 4 reflects quarters 2, 3, 4, 5, and 6 but is not affected by quarter 1. Similarly, the moving average for quarter 5 forgets quarters 1 and 2. Both of these problems are addressed by **exponential smoothing**.

Exponentially Smoothed Time Series

$$S_t = wy_t + (1 - w)S_{t-1} \text{ for } t \geq 2$$

where

S_t = Exponentially smoothed time series at time period t

y_t = Time series at time period t

S_{t-1} = Exponentially smoothed time series at time period $t - 1$

w = Smoothing constant, where $0 \leq w \leq 1$

We begin by setting

$$S_1 = y_1$$

Then

$$\begin{aligned} S_2 &= wy_2 + (1 - w)S_1 \\ &= wy_2 + (1 - w)y_1 \\ S_3 &= wy_3 + (1 - w)S_2 \\ &= wy_3 + (1 - w)[wy_2 + (1 - w)y_1] \\ &= wy_3 + w(1 - w)y_2 + (1 - w)^2 y_1 \end{aligned}$$

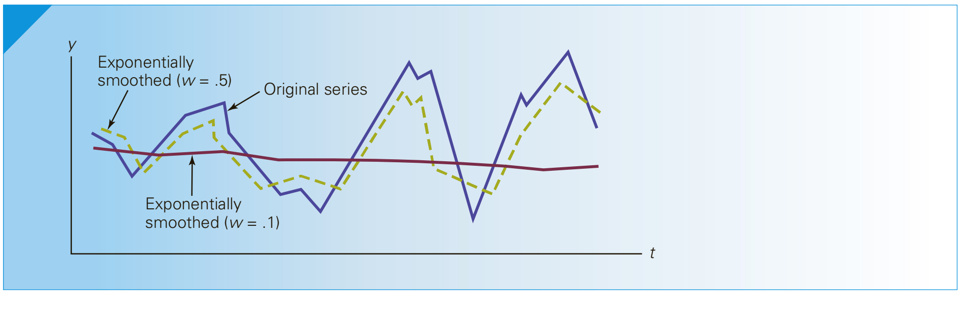
and so on. In general, we have

$$S_t = wy_t + w(1 - w)y_{t-1} + w(1 - w)^2 y_{t-2} + \cdots + (1 - w)^{t-1} y_1$$

This formula states that the smoothed time series in period t depends on all the previous observations of the time series.

The smoothing constant w is chosen on the basis of how much smoothing is required. A small value of w produces a great deal of smoothing. A large value of w results in very little smoothing. Figure 20.6 depicts a time series and two exponentially smoothed series with $w = .1$ and $w = .5$.

FIGURE **20.6** Time Series and Two Exponentially Smoothed Series



Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXAMPLE 20.1

Gasoline Sales, Part 2

Apply the exponential smoothing technique with $w = .2$ and $w = .7$ to the data in Example 20.1, and graph the results.

SOLUTION:

COMPUTE

MANUALLY:

The exponentially smoothed values are calculated from the formula

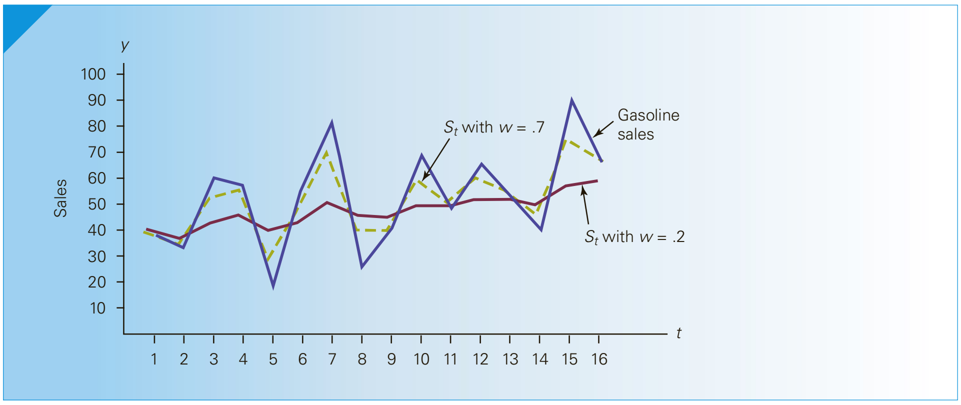
$$S_t = wy_t + (1 - w)S_{t-1}$$

The results with $w = .2$ and $w = .7$ are shown in the following table.

Time Period	Gasoline Sales	Exponentially Smoothed with $w = .2$	Exponentially Smoothed with $w = .7$
1	39	39.0	39.0
2	37	38.6	37.6
3	61	43.1	54.0
4	58	46.1	56.8
5	18	40.5	29.6
6	56	43.6	48.1
7	82	51.2	71.8
8	27	46.4	40.4
9	41	45.3	40.8
10	69	50.1	60.6
11	49	49.8	52.5
12	66	53.1	61.9
13	54	53.3	56.4
14	42	51.0	46.3
15	90	58.8	76.9
16	66	60.2	69.3

Figure 20.7 shows the exponentially smoothed time series.

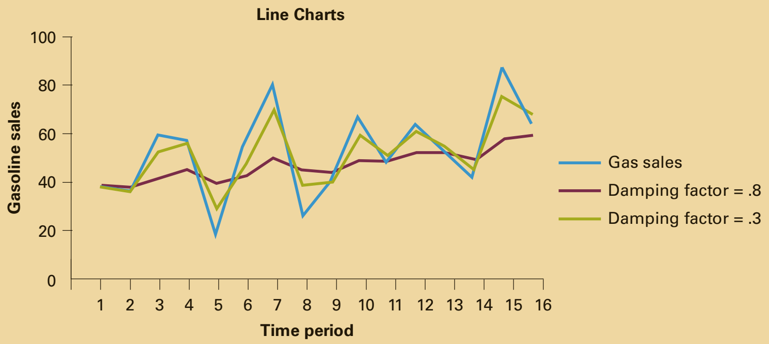
FIGURE 20.7 Quarterly Gasoline Sales and Exponentially Smoothed Sales with $w = .2$ and $w = .7$



Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXCEL Data Analysis

	A	B	C
1	Gas Sales	Damping factor = .8	Damping factor = .3
2	39	39.0	39.0
3	37	38.6	37.6
4	61	43.1	54.0
5	58	46.1	56.8
6	18	40.5	29.6
7	56	43.6	48.1
8	82	51.2	71.8
9	27	46.4	40.4
10	41	45.3	40.8
11	69	50.1	60.6
12	49	49.8	52.5
13	66	53.1	61.9
14	54	53.3	56.4
15	42	51.0	46.3
16	90	58.8	76.9
17	66	60.2	69.3



INSTRUCTIONS

1. Type or import the data into one column (Open Xm20-01.)
2. Click **Data, Data Analysis,** and **Exponential Smoothing**.
3. Specify the **Input Range** (A1:A17). Type the **Damping factor**, which is 1 − w (.8). Specify the **Output Range** (B1). To calculate the second exponentially smoothed time series specify 1 − w (.3) **Output Range** (C1).

To modify the table so that the smoothed values appear the way we calculated, manually click the cell containing the last smoothed value displayed here (58.8) and drag it to the cell below to reveal the final smoothed value (60.2 and 69.3).

INTERPRET

Figure 20.7 depicts the graph of the original time series and the exponentially smoothed series. As you can see, $w = .7$ results in very little smoothing, whereas $w = .2$ results in perhaps too much smoothing. In both smoothed time series, it is difficult to discern the seasonal pattern that we detected by using moving averages. A different value of w (perhaps $w = .5$) would be likely to produce more satisfactory results.

Moving averages and exponential smoothing are relatively crude methods of removing the random variation to discover the existence of other components. In the next section, we attempt to measure these components more precisely.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXERCISES

20.1 Xr20-01 For the following time series, compute the three-period moving averages.

Period	Time Series	Period	Time Series
1	48	7	43
2	41	8	52
3	37	9	60
4	32	10	48
5	36	11	41
6	31	12	30

20.2 Compute the five-period moving averages for the time series in Exercise 20.1.

20.3 For Exercises 20.1 and 20.2, graph the time series and the two moving averages.

20.4 Xr20-04 For the following time series, compute the three-period moving averages.

Period	Time Series	Period	Time Series
1	16	7	24
2	22	8	29
3	19	9	21
4	24	10	23
5	30	11	19
6	26	12	15

20.5 For Exercise 20.4, compute the five-period moving averages.

20.6 For Exercises 20.4 and 20.5, graph the time series and the two moving averages.

20.7 Xr20-07 Apply exponential smoothing with $w = .1$ to help detect the components of the following time series.

Period	1	2	3	4	5
Time Series	12	18	16	24	17
Period	6	7	8	9	10
Time Series	16	25	21	23	14

20.8 Repeat Exercise 20.7 with $w = .8$.

20.9 For Exercises 20.7 and 20.8, draw the time series and the two sets of exponentially smoothed values. Does there appear to be a trend component in the time series?

20.10 Xr20-10 Apply exponential smoothing with $w = .1$ to help detect the components of the following time series.

Period	1	2	3	4	5
Time Series	38	43	42	45	46
Period	6	7	8	9	10
Time Series	48	50	49	46	45

20.11 Repeat Exercise 20.10 with $w = .8$.

20.12 For Exercises 20.10 and 20.11, draw the time series and the two sets of exponentially smoothed values. Does there appear to be a trend component in the time series?

20.13 Xr20-13 The following daily sales figures have been recorded in a medium-size merchandising firm.

	Week			
Day	1	2	3	4
Monday	43	51	40	64
Tuesday	45	41	57	58
Wednesday	22	37	30	33
Thursday	25	22	33	38
Friday	31	25	37	25

a. Compute the three-day moving averages.
b. Plot the time series and the moving averages on a graph.
c. Does there appear to be a seasonal (weekly) pattern?

20.14 For Exercise 20.13, compute the five-day moving averages, and superimpose these on the same graph. Does this help you answer part (c) of Exercise 20.13?

20.15 Xr20-15 The following quarterly sales of a department store chain were recorded for the years 2013–2016.

	Year			
Quarter	2013	2014	2015	2016
1	18	33	25	41
2	22	20	36	33
3	27	38	44	52
4	31	26	29	45

a. Calculate the four-quarter centered moving averages.
b. Graph the time series and the moving averages.
c. What can you conclude from your time-series smoothing?

20.16 Repeat Exercise 20.15, using exponential smoothing with $w = .4$.

20.17 Repeat Exercise 20.15, using exponential smoothing with $w = .8$.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

20-3 Trend and Seasonal Effects

In the previous section, we described how smoothing a time series can give us a clearer picture of which components are present. In order to forecast, however, we often need more precise measurements of the time-series components.

20-3a Trend Analysis

A trend can be linear or nonlinear and, indeed, can take on a whole host of functional forms. The easiest way of measuring the long-term trend is by regression analysis, where the independent variable is time. If we believe that the long-term trend is approximately linear, we will use the linear model introduced in Chapter 16:

$$y = \beta_0 + \beta_1 t + \varepsilon$$

If we believe that the trend is nonlinear, we can use one of the polynomial models described in Chapter 18. For example, the quadratic model is

$$y = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon$$

In most realistic applications, the linear model is used. We will demonstrate how the long-term trend is measured and applied later in this section.

20-3b Seasonal Analysis

Seasonal variation may occur within a year or within shorter intervals, such as a month, week, or day. To measure the seasonal effect, we compute seasonal indexes, which gauge the degree to which the seasons differ from one another. One requirement necessary to calculate seasonal indexes is a time series sufficiently long enough to allow us to observe the variable over several seasons. For example, if the seasons are defined as the quarters of a year, we need to observe the time series for at least 4 years. The **seasonal indexes** are computed in the following way.

20-3c Procedure for Computing Seasonal Indexes

1. Remove the effect of seasonal and random variation by regression analysis; that is, compute the sample regression line

$$\hat{y}_t = b_0 + b_1 t$$

2. For each time period compute the ratio

$$\frac{y_t}{\hat{y}_t}$$

This ratio removes most of the trend variation.

3. For each type of season, compute the average of the ratios in step 2. This procedure removes most (but seldom all) of the random variation, leaving a measure of seasonality.

4. Adjust the averages in step 3 so that the average of all the seasons is 1 (if necessary).

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXAMPLE 20.3

DATA
Xm20-03

Hotel Quarterly Occupancy Rates

The tourist industry is subject to seasonal variation. In most resorts, the spring and summer seasons are considered the "high" seasons. Fall and winter (except for Christmas and New Year's) are "low" seasons. A hotel in Bermuda has recorded the occupancy rate for each quarter for the past 5 years. These data are shown here. Measure the seasonal variation by computing the seasonal indexes.

Year	Quarter	Occupancy Rate
2012	1	.561
	2	.702
	3	.800
	4	.568
2013	1	.575
	2	.738
	3	.868
	4	.605
2014	1	.594
	2	.738
	3	.729
	4	.600
2015	1	.622
	2	.708
	3	.806
	4	.632
2016	1	.665
	2	.835
	3	.873
	4	.670

SOLUTION:

COMPUTE

MANUALLY:

We performed a regression analysis with $y = \text{occupancy rate}$ and $t = \text{time period}$ $1, 2, \ldots, 20$. The regression equation is

$$\hat{y} = .639368 + .005246t$$

For each time period, we computed the ratio

$$\frac{y_t}{\hat{y}_t}$$

In the next step, we collected the ratios associated with each quarter and computed the average. We then computed the seasonal indexes by adjusting the average ratios so that they summed to 4.0, if necessary. In this example, it was not necessary.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Year	Quarter	t	y_t	$\hat{y} = .639368 + .005246t$	Ratio $\frac{y_t}{\hat{y}_t}$
2012	1	1	.561	.645	.870
	2	2	.702	.650	1.080
	3	3	.800	.655	1.221
	4	4	.568	.660	.860
2013	1	5	.575	.666	.864
	2	6	.738	.671	1.100
	3	7	.868	.676	1.284
	4	8	.605	.681	.888
2014	1	9	.594	.687	.865
	2	10	.738	.692	1.067
	3	11	.729	.697	1.046
	4	12	.600	.702	.854
2015	1	13	.622	.708	.879
	2	14	.708	.713	.993
	3	15	.806	.718	1.122
	4	16	.632	.723	.874
2016	1	17	.665	.729	.913
	2	18	.835	.734	1.138
	3	19	.873	.739	1.181
	4	20	.670	.744	.900

	Quarter			
Year	1	2	3	4
2012	.870	1.080	1.221	.860
2013	.864	1.100	1.284	.888
2014	.865	1.067	1.046	.854
2015	.879	.993	1.122	.874
2016	.913	1.138	1.181	.900
Average	.878	1.076	1.171	.875
Index	.878	1.076	1.171	.875

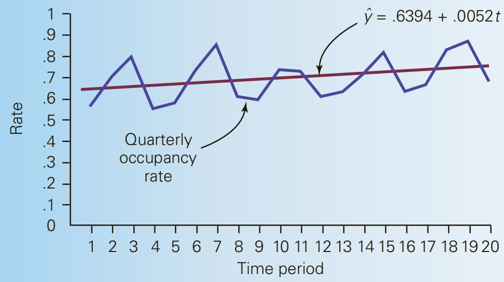
INTERPRET

The seasonal indexes tell us that, on average, the occupancy rates in the first and fourth quarters are below the annual average, and the occupancy rates in the second and third quarters are above the annual average. We expect the occupancy rate in the first quarter to be 12.2%(100% – 87.8%) below the annual rate. The second and third quarters' rates are expected to be 7.6% and 17.1%, respectively, above the annual rate. The fourth quarter's rate is 12.5% below the annual rate.

Figure 20.8 depicts the time series and the regression trend line.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

FIGURE **20.8** Time Series and Trend for Example 20.3



20-3d Deseasonalizing a Time Series

One application of seasonal indexes is to remove the seasonal variation in a time series. The process is called **deseasonalizing**, and the result is called a **seasonally adjusted time series**. Often this allows the statistics practitioner to more easily compare the time series across seasons. For example, the unemployment rate varies according to the season. During the winter months, unemployment usually rises; it falls in the spring and summer. The seasonally adjusted unemployment rate allows economists to determine whether unemployment has increased or decreased over the previous months. The process is easy: Simply divide the time series by the seasonal indexes. To illustrate, we have deseasonalized the occupancy rates in Example 20.3. The results are shown next.

Year	Quarter	Occupancy Rate y_t	Seasonal Index	Seasonally Adjusted Occupancy Rate
2012	1	.561	.878	.639
	2	.702	1.076	.652
	3	.800	1.171	.683
	4	.568	.875	.649
2013	1	.575	.878	.655
	2	.738	1.076	.686
	3	.868	1.171	.741
	4	.605	.875	.691
2014	1	.594	.878	.677
	2	.738	1.076	.686
	3	.729	1.171	.623
	4	.600	.875	.686
2015	1	.622	.878	.708
	2	.708	1.076	.658
	3	.806	1.171	.688
	4	.632	.875	.722
2016	1	.665	.878	.757
	2	.835	1.076	.776
	3	.873	1.171	.746
	4	.670	.875	.766

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

By removing the seasonality, we can see when there has been a "real" increase or decrease in the occupancy rate. This enables the statistics practitioner to examine the factors that produced the rate change. We can more easily see that there has been an increase in the occupancy rate over the 5-year period.

In the next section, we show how to forecast with seasonal indexes.

EXERCISES

20.18 Xr20-18 Plot the following time series. Would the linear or quadratic model fit better?

Period	1	2	3	4	5	6	7	8
Time Series	.5	.6	1.3	2.7	4.1	6.9	10.8	19.2

20.19 Xr20-19 Plot the following time series to determine which of the trend models appears to fit better.

Period	1	2	3	4	5
Time Series	55	57	53	49	47

Period	6	7	8	9	10
Time Series	39	41	33	28	20

20.20 Refer to Exercise 20.18. Use regression analysis to calculate the linear and quadratic trends. Which line fits better?

20.21 Refer to Exercise 20.19. Use regression analysis to calculate the linear and quadratic trends. Which line fits better?

20.22 Xr20-22 For the following time series, compute the seasonal (daily) indexes.

The regression line is

$$\hat{y} = 16.8 + .366t \quad (t = 1, 2, \ldots, 20)$$

	Week			
Day	**1**	**2**	**3**	**4**
Monday	12	11	14	17
Tuesday	18	17	16	21
Wednesday	16	19	16	20
Thursday	25	24	28	24
Friday	31	27	25	32

20.23 Xr20-23 Given the following time series, compute the seasonal indexes.

The regression equation is

$$\hat{y} = 47.7 - 1.06t \quad (t = 1, 2, \ldots, 20)$$

	Year				
Quarter	**1**	**2**	**3**	**4**	**5**
1	55	41	43	36	50
2	44	38	39	32	25
3	46	37	39	30	24
4	39	30	35	25	22

Applications

20.24 Xr20-24 The quarterly earnings (in $millions) of a large soft-drink manufacturer have been recorded for the years 2013–2016. These data are listed here. Compute the seasonal indexes given the regression line

$$\hat{y} = 61.75 + 1.18t \quad (t = 1, 2, \ldots, 16)$$

	Year			
Quarter	**2013**	**2014**	**2015**	**2016**
1	52	57	60	66
2	67	75	77	82
3	85	90	94	98
4	54	61	63	67

The following exercises require a computer and software.

20.25 Xr20-25 College and university enrollment increased sharply during the 1970s and 1980s. However, since then, the rate of growth has slowed. To help forecast future enrollments, an economist recorded the total U.S. college and university enrollment from 1993 to 2009. These data (in thousands) are listed here.

Year	1993	1994	1995	1996	1997	1998	1999
Enrollment	13,898	15,022	14,715	15,226	15,436	15,546	15,203

Year	2000	2001	2002	2003	2004	2005	2006
Enrollment	15,314	15,873	16,497	16,638	17,272	17,487	17,672

Year	2007	2008	2009
Enrollment	18,248	19,103	20,428

Source: Statistical Abstract of the United States, 2009, Table 279.

a. Plot the time series
b. Use regression analysis to determine the trend.

20.26 Xr20-26 Foreign trade is important to the United States. No country exports and imports more. However, there has been a large trade imbalance in many sectors. To measure the extent of the problem, an economist recorded the difference between exports and imports of merchandise (excluding military) for the years 1980 to 2012.

a. Plot the trade balance.
b. Apply regression analysis to measure the trend.

Source: Federal Reserve St. Louis.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

20.27 Xr20-27 The number of cable television subscribers has increased over the past 5 years. The marketing manager for a cable company has recorded the numbers of subscribers for the past 24 quarters.

a. Plot the numbers.

b. Compute the seasonal (quarterly) indexes.

20.28 Xr20-28 The owner of a pizzeria wants to forecast the number of pizzas she will sell each day. She recorded the numbers sold daily for the past 4 weeks. Calculate the seasonal (daily) indexes.

20.29 Xr20-29 A manufacturer of ski equipment is in the process of reviewing his accounts receivable. He noticed that there appears to be a seasonal pattern with the accounts receivable increasing in the winter months and decreasing during the summer. The quarterly accounts receivable (in $millions) were recorded. Compute the seasonal (quarterly) indexes.

20-4 Introduction to Forecasting

Many different forecasting methods are available for the statistics practitioner. One factor to be considered in choosing among them is the type of component that makes up the time series. Even then, however, we have several different methods from which to choose. One way of deciding which method to apply is to select the technique that achieves the greatest forecast accuracy. The most commonly used measures of forecast accuracy are **mean absolute deviation (MAD)** and the **sum of squares for forecast errors (SSE)**.

Mean Absolute Deviation

$$\text{MAD} = \frac{\sum_{i=1}^{n} |y_t - F_t|}{n}$$

where

y_t = Actual value of the time series at time period t

F_t = Forecasted value of the time series at time period t

n = Number of time periods

Sum of Squares for Forecast Error

$$\text{SSE} = \sum_{i=1}^{n} (y_t - F_t)^2$$

MAD averages the absolute differences between the actual and forecast values; SSE is the sum of the squared differences. Which measure to use in judging forecast accuracy depends on the circumstances. If avoiding large errors is important, SSE should be used because it penalizes large deviations more heavily than does MAD. Otherwise, use MAD.

It is probably best to use some of the observations of the time series to develop several competing forecasting models and then forecast for the remaining time periods. Afterward, compute MAD or SSE for the forecasts. For example, if we have 5 years of monthly observations, use the first 4 years to develop the forecasting models and then use them to forecast the fifth year. Because we know the actual values in the fifth year, we can choose the technique that results in the most accurate forecast using either MAD or SSE.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXAMPLE 20.4 Comparing Forecasting Models

Annual data from 1976 to 2012 were used to develop three different forecasting models. Each model was used to forecast the time series for 2013, 2014, 2015, and 2016. The forecasted and actual values for these years are shown here. Use MAD and SSE to determine which model performed best.

Year	Actual Time Series	1	2	3
2013	129	136	118	130
2014	142	148	141	146
2015	156	150	158	170
2016	183	175	163	180

SOLUTION:

For model 1, we have

$$\text{MAD} = \frac{|129 - 136| + |142 - 148| + |156 - 150| + |183 - 175|}{4}$$

$$= \frac{7 + 6 + 6 + 8}{4} = 6.75$$

$$\text{SSE} = (129 - 136)^2 + (142 - 148)^2 + (156 - 150)^2 + (183 - 175)^2$$

$$= 49 + 36 + 36 + 64 = 185$$

For model 2, we compute

$$\text{MAD} = \frac{|129 - 118| + |142 - 141| + |156 - 158| + |183 - 163|}{4}$$

$$= \frac{11 + 1 + 2 + 20}{4} = 8.5$$

$$\text{SSE} = (129 - 118)^2 + (142 - 141)^2 + (156 - 158)^2 + (183 - 163)^2$$

$$= 121 + 1 + 4 + 400 = 526$$

The measures of forecast accuracy for model 3 are

$$\text{MAD} = \frac{|129 - 130| + |142 - 146| + |156 - 170| + |183 - 180|}{4}$$

$$= \frac{1 + 4 + 14 + 3}{4} = 5.5$$

$$\text{SSE} = (129 - 130)^2 + (142 - 146)^2 + (156 - 170)^2 + (183 - 180)^2$$

$$= 1 + 16 + 196 + 9 = 222$$

Model 2 is inferior to both models 1 and 3, no matter how we measure forecast accuracy. Using MAD, model 3 is best—but using SSE, model 1 is most accurate. The choice between model 1 and model 3 should be made on the basis of whether we prefer a model that consistently produces moderately accurate forecasts (model 1) or one whose forecasts come quite close to most actual values but miss badly in a small number of time periods (model 3).

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXERCISES

20.30 For the actual and forecast values of a time series shown here, calculate MAD and SSE.

Period	1	2	3	4	5
Forecast	173	186	192	211	223
Actual Value	166	179	195	214	220

20.31 Two forecasting models were used to predict the future values of a time series. These are shown here together with the actual values. Compute MAD and SSE for each model to determine which was more accurate.

Period	1	2	3	4
Forecast (Model 1)	7.5	6.3	5.4	8.2
Forecast (Model 2)	6.3	6.7	7.1	7.5
Actual	6.0	6.6	7.3	9.4

20.32 Calculate MAD and SSE for the forecasts that follow.

Period	1	2	3	4	5
Forecast	63	72	86	71	60
Actual	57	60	70	75	70

20.33 Three forecasting techniques were used to predict the values of a time series. These values are given in the following table. Compute MAD and SSE for each technique to determine which was most accurate.

Period	1	2	3	4	5
Forecast (Model 1)	21	27	29	31	35
Forecast (Model 2)	22	24	26	28	30
Forecast (Model 3)	17	20	25	31	39
Actual	19	24	28	32	38

20-5 FORECASTING MODELS

There is a large number of different forecasting techniques available to statistics practitioners. However, many are beyond the level of this book. In this section, we present three models. Similar to the method of choosing the correct statistical inference technique in Chapters 12 to 19, the choice of model depends on the time-series components.

20-5a Forecasting with Exponential Smoothing

If the time series displays a gradual trend or no trend and no evidence of seasonal variation, exponential smoothing can be effective as a forecasting method. Suppose that t represents the most recent time period and we've computed the exponentially smoothed value S_t. This value is then the forecasted value at time $t + 1$; that is,

$$F_{t+1} = S_t$$

If we wish, we can forecast two or three or any number of periods into the future:

$$F_{t+2} = S_t \qquad \text{or} \qquad F_{t+3} = S_t$$

It must be understood that the accuracy of the forecast decreases rapidly for predictions more than one time period into the future. However, as long as we're dealing with time series with no cyclical or seasonal variation, we can produce reasonably accurate predictions for the next time period.

20-5b Forecasting with Seasonal Indexes

If the time series is composed of seasonal variation and long-term trend, we can use seasonal indexes and the regression equation to forecast.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Forecast of Trend and Seasonality

The forecast for time period t is

$$F_t = [b_0 + b_1 t] \times SI_t$$

where

$$F_t = \text{Forecast for period } t$$
$$b_0 + b_1 t = \text{Regression equation}$$
$$SI_t = \text{Seasonal index for period } t$$

EXAMPLE 20.5

Forecasting Hotel Occupancy Rates

Forecast hotel occupancy rates for next year in Example 20.3.

SOLUTION:

In the process of computing the seasonal indexes, we computed the trend line. It is

$$\hat{y} = .639 + .00525t$$

For $t = 21, 22, 23$, and 24, we calculate the forecasted trend values.

Quarter	t	$\hat{y} = .639 + .00525t$
1	21	$.639 + .00525(21) = .749$
2	22	$.639 + .00525(22) = .755$
3	23	$.639 + .00525(23) = .760$
4	24	$.639 + .00525(24) = .765$

We now multiply the forecasted trend values by the seasonal indexes calculated in Example 20.3. The seasonalized forecasts are as follows:

Quarter	t	Trend Value $\hat{y}_t$	Seasonal Index	Forecast $F_t = \hat{y}_t \times SI_t$
1	21	.749	.878	$.749 \times .878 = .658$
2	22	.755	1.076	$.755 \times 1.076 = .812$
3	23	.760	1.171	$.760 \times 1.171 = .890$
4	24	.765	.875	$.765 \times .875 = .670$

INTERPRET

We forecast that the quarterly occupancy rates during the next year will be .658, .812, .890, and .670.

20-5c Autoregressive Model

In Chapter 17, we discussed autocorrelation wherein the errors are not independent of one another. The existence of strong autocorrelation indicates that the model has been misspecified, which usually means that until we improve the regression model,

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

it will not provide an adequate fit. However, autocorrelation also provides us with an opportunity to develop another forecasting technique. If there is no obvious trend or seasonality and we believe that there is a correlation between consecutive residuals, the **autoregressive model** may be most effective.

Autoregressive Forecasting Model

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon$$

The model specifies that consecutive values of the time series are correlated. We estimate the coefficient in the usual way. The estimated regression line is defined as

$$\hat{y}_t = b_0 + b_1 y_{t-1}$$

EXAMPLE 20.6

DATA Xm20-06

Forecasting Changes to the Consumer Price Index

The consumer price index (CPI) is used as a general measure of inflation. It is an important measure because a high rate of inflation often influences governments to take corrective measures. The table below lists the consumer price index from 1980 to 2015 and the annual percentage increases in the CPI. Forecast next year's change in the CPI.

Year	CPI	% Change	Year	CPI	% Change
1980	82.4		1998	163.0	1.55%
1981	90.9	10.38%	1999	166.6	2.19%
1982	96.5	6.16%	2000	172.2	3.37%
1983	99.6	3.16%	2001	177.0	2.82%
1984	103.9	4.37%	2002	179.9	1.60%
1985	107.6	3.53%	2003	184.0	2.30%
1986	109.7	1.94%	2004	188.9	2.67%
1987	113.6	3.58%	2005	195.3	3.37%
1988	118.3	4.10%	2006	201.6	3.22%
1989	123.9	4.79%	2007	207.3	2.87%
1990	130.7	5.42%	2008	215.3	3.81%
1991	136.2	4.22%	2009	214.6	-0.32%
1992	140.3	3.04%	2010	218.1	1.64%
1993	144.5	2.97%	2011	224.9	3.14%
1994	148.2	2.60%	2012	229.6	2.08%
1995	152.4	2.81%	2013	233.0	1.47%
1996	156.9	2.94%	2014	236.7	1.61%
1997	160.5	2.34%	2015	237.0	0.12%

Source: U.S. Bureau of Labor Statistics.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

SOLUTION:

Notice that we included the CPI for 1980 because we wanted to determine the percentage change for 1981. We will use the percentage changes for 1981 to 2014 as the independent variable and the percentage change from 1982 to 2015 as the dependent variable. File Xm20-06 stores the data in the format necessary to determine the autoregressive model.

EXCEL Data Analysis

	A	B	C	D	E
1		Coefficients	Standard Error	t Stat	P-value
2	Intercept	0.0145	0.0038	3.78	0.0007
3	% Change	0.4481	0.1058	4.23	0.0002

INTERPRET

The regression line is

$$\hat{y}_t = .0145 + .4481 y_{t-1}$$

Because the last CPI change is $.12\%$, our forecast for 2016 is

$$\begin{aligned}\hat{y}_{2016} &= .0145 + .4481 y_{2015} \\ &= .0145 + .4481(.12\%) = .07\%\end{aligned}$$

The autoregressive model forecasts a $.07\%$ increase in the CPI for the year 2016.

Housing Starts: Solution


iStockphoto.com/stocknroll

A preliminary examination of the data reveals that there is a very small upward trend over the 5-year period. Moreover, the number of housing starts varies by month. The presence of these components suggests that we determine the linear trend and seasonal (monthly) indexes.

With housing starts as the dependent variable and the month as the independent variable, Excel yielded the following regression line:

$$\hat{y} = 11.46 + .0808t \qquad t = 1, 2, \ldots, 60$$

The seasonal indexes were computed as follows.

Season	Index
1	.5974
2	.6548
3	.9800
4	1.0697
5	1.1110
6	1.1917
7	1.2050
8	1.2276
9	1.0960
10	1.0226
11	.9960
12	.8483

(*Continued*)

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

The regression equation was used again to predict the number of housing starts based on the linear trend:

$$\hat{y} = 11.46 + .0808t \qquad t = 61, 62, \ldots, 72$$

These figures were multiplied by the seasonal indexes, which resulted in the following forecasts.

Period	Month	$\hat{y} = 11.46 + .0808t$	Seasonal Index	Forecasts
61	January	16.39	.5974	9.79
62	February	16.47	.6548	10.79
63	March	16.55	.9800	16.22
64	April	16.63	1.0697	17.79
65	May	16.71	1.1110	18.57
66	June	16.79	1.1917	20.01
67	July	16.87	1.2050	20.33
68	August	16.95	1.2276	20.81
69	September	17.04	1.0960	18.67
70	October	17.12	1.0226	17.50
71	November	17.20	.9960	17.13
72	December	17.28	.8483	14.66

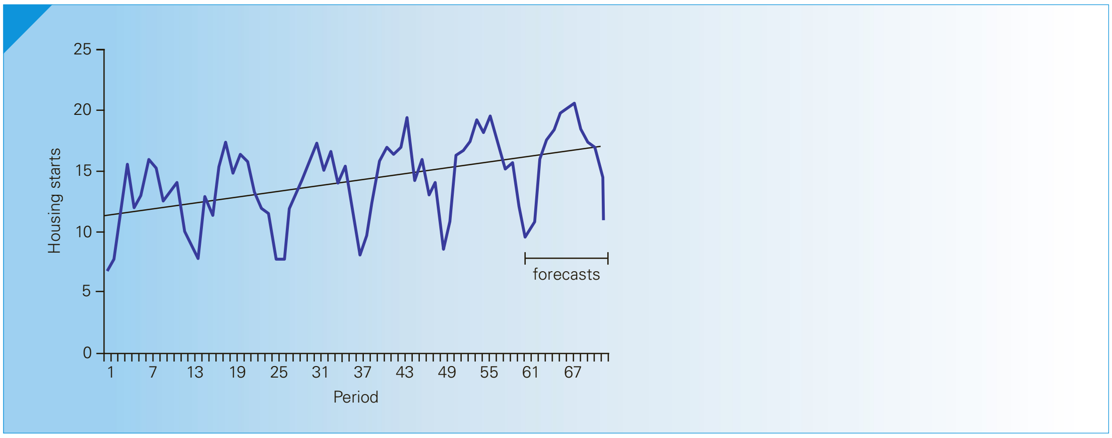
This table displays the actual and forecasted housing starts for 2006. Figure 20.9 depicts the time series, trend line, and forecasts.

Period	Month	Forecasts	Actual
61	January	9.79	13.3
62	February	10.79	10.1
63	March	16.22	12.9
64	April	17.79	16.0
65	May	18.57	18.8
66	June	20.01	16.1
67	July	20.33	13.7
68	August	20.81	15.6
69	September	18.67	12.3
70	October	17.50	13.3
71	November	17.13	12.2
72	December	14.66	12.9

The size of the error was measured by MAD and SSE. They are

$$\text{MAD} = 42.55$$
$$\text{SSE} = 199.13$$

FIGURE **20.9** **Time Series, Trend, and Forecasts of Housing Starts**



Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

EXERCISES

20.34 The following trend line and seasonal indexes were computed from 10 years of quarterly observations. Forecast the next year's time series.

$$\hat{y} = 150 + 3t \qquad t = 1, 2, \ldots, 40$$

Quarter	Seasonal Index
1	.7
2	1.2
3	1.5
4	.6

20.35 The following trend line and seasonal indexes were computed from 4 weeks of daily observations. Forecast the 7 values for next week.

$$\hat{y} = 120 + 2.3t \qquad t = 1, 2, \ldots, 28$$

Day	Seasonal Index
Sunday	1.5
Monday	.4
Tuesday	.5
Wednesday	.6
Thursday	.7
Friday	1.4
Saturday	1.9

20.36 Use the following autoregressive equation to forecast the next value of the time series if the last observed value is 65.

$$\hat{y} = 625 - 1.3y_{t-1}$$

20.37 The following autoregressive equation was developed. Forecast the next value if the last observed value was 11.

$$\hat{y} = 155 + 21y_{t-1}$$

20.38 Apply exponential smoothing with $w = .4$ to forecast the next four quarters in Exercise 20.15.

20.39 Use the seasonal indexes and trend line to forecast the time series for the next 5 days in Exercise 20.22.

20.40 Refer to Exercise 20.23. Use the seasonal indexes and the trend line to forecast the time series for the next four quarters.

Applications

20.41 Use the seasonal indexes and trend line to forecast the quarterly earnings for the years 2014 and 2015 in Exercise 20.24.

20.42 Refer to Exercise 20.25. Forecast next year's enrollment using the following methods.
a. Autoregressive forecasting model.
b. Exponential smoothing method with $w = .5$.

20.43 Refer to Exercise 20.26. Forecast next year's merchandise trade balance using the following methods.
a. Autoregressive forecasting model.
b. Exponential smoothing method with $w = .7$.

20.44 Use the seasonal indexes and trend line from Exercise 20.27 to forecast the number of cable subscribers for the next four quarters.

20.45 Refer to Exercise 20.28. Use the seasonal indexes and trend line to forecast the number of pizzas to be sold for each of the next 7 days.

20.46 Apply the trend line and seasonal indexes from Exercise 20.29 to forecast accounts receivable for the next four quarters.

Exercises 20.47–20.51 are based on the following problem.

Xr20-47 The revenues (in $millions) of a chain of ice cream stores are listed for each quarter during the previous 5 years.

	Year				
Quarter	2012	2013	2014	2015	2016
1	16	14	17	18	21
2	25	27	31	29	30
3	31	32	40	45	52
4	24	23	27	24	32

20.47 Plot the time series.

20.48 Discuss why exponential smoothing is not recommended as a forecasting tool in this problem.

20.49 Use regression analysis to determine the trend line.

20.50 Determine the seasonal indexes.

20.51 Using the seasonal indexes and trend line, forecast revenues for the next four quarters.

20.52 Xr20-52 The number of housing starts (in 1,000s) in the northeast United States for the years 2004 to 2009 were recorded.
a. Use the 2004–2008 data to calculate the seasonal indexes.
b. Use the indexes and regression analysis to forecast the number of housing starts in 2009.
c. Calculate SSE and MAD to measure how well (or poorly) the forecasts fared.

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203

Chapter Summary

In this chapter, we discussed the classical time series and its decomposition into trend, seasonal, and random variation. Moving averages and exponential smoothing were used to remove some of the random variation, making it easier to detect trend and seasonality. The long-term trend was measured by regression analysis. Seasonal variation was measured by computing the seasonal indexes. Three forecasting techniques were described in this chapter: exponential smoothing, forecasting with seasonal indexes, and the autoregressive model.

IMPORTANT TERMS:

SYMBOLS:

Symbol	Represents
y_y	Time series
S_t	Exponentially smoothed time series
w	Smoothing constant
F_t	Forecasted time series

FORMULAS:

Exponential smoothing

$$S_t = wy_t + (1 - w)S_{t-1}$$

Mean absolute deviation

$$\text{MAD} = \frac{\sum_{i=1}^{n} |y_t - F_t|}{n}$$

Sum of squares for error

$$SSE = \sum_{i=1}^{n} (y_t - F_t)^2$$

Forecast of trend and seasonality

$$F_t = [b_0 + b_1 t] \times SI_t$$

Autoregressive model

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon$$

COMPUTER INSTRUCTIONS:

Technique	Excel
Moving averages	837
Exponential smoothing	841

Copyright 2018 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. WCN 02-200-203