The farmer's problem - 2

After thinking about this solution, the farmer becomes worried. He has indeed experienced quite different yields for the same crop over different years mainly because of changing weather conditions. Most crops need rain during the few weeks after seeding or planting, then sunshine is welcome for the rest of the growing period. Sunshine should, however, not turn into drought, which causes severe yield reductions. Dry weather is again beneficial during harvest. From all these factors, yields varying 20 to 25% above or below the mean yield are not unusual.

A Scenario Representation to The Farmer's Problem

A first possibility is to assume some correlation among the yields of the different crops. A very simplified representation of this would be to assume that years are good, fair, or bad for all crops, resulting in above average, average, or below average yields for all crops. To fix these ideas, "above" and "below" average indicate a yield 20% above or below the mean yield given in Table 1. For simplicity, we assume that weather conditions and yields for the farmer do not have a significant impact on prices.

Long-term weather forecasts would be very helpful here. Unfortunately, as even meteorologists agree, weather conditions cannot be accurately predicted six months ahead. The farmer must make up his mind without perfect information on yields.

The main issue here is clearly on sugar beet production. Planting large surfaces would make it certain to produce and sell the quota, but would also make it likely to sell some sugar beets at the unfavorable price. Planting small surfaces would make it likely to miss the opportunity to sell the full quota at the favorable price.

The farmer now realizes that he is unable to make a perfect decision that would be best in all circumstances. He would, therefore, want to assess the benefits and losses of each decision in each situation. Decisions on land assignment ($x1$,$x2$,$x3$) have to be taken now, but sales and purchases (w_i, $i = 1,...,4$, y_j, $j = 1,2$) depend on the yields. It is useful to index those decisions by a scenario index $s = 1,2,3$ corresponding to above average, average, or below average yields, respectively. This creates a new set of variables of the form w_{is} , $i = 1,2,3,4$, $s = 1,2,3$ and y_{js} , $j = 1,2$, $s = 1,2,3$. As an example, w_{32} represents the amount of sugar beets sold at the favorable price if yields are average.

Assuming the farmer wants to maximize long-run profit, it is reasonable for him to seek a solution that maximizes his expected profit (this assumption means that the farmer is neutral about risk). If the three scenarios have an equal probability of 1/3, the farmer's problem reads as follows:

Table 5 Optimal solution based on the stochastic model (1.2).

		Wheat	Corn	Sugar Beets
First Stage	Area (acres)	170	80	250
$s = 1$ Above	Yield (T)	510	288	6000
	Sales (T)	310	48	6000 (favor. price)
	Purchase (T)	–	–	–
$s = 2$ Average	Yield (T)	425	240	5000
	Sales (T)	225	–	5000 (favor. price)
	Purchase (T)	–	–	–
$s = 3$ Below	Yield (T)	340	192	4000
	Sales (T)	140	–	4000 (favor. price)
	Purchase (T)	–	48	–
Overall profit: $108,390				

This solution illustrates that it is impossible, under uncertainty, to find a solution that is ideal under all circumstances. Selling some sugar beets at the unfavorable price or having some unused quota is a decision that would never take place with a perfect forecast. Such decisions can appear in a stochastic model because decisions have to be balanced or hedged against the various scenarios.