**Chapter 25 - Bernoulli’s Errors**

One hundred years before Fechner, Bernoulli invented psychophysics to explain this aversion to risk. His idea was straightforward: people’s choices are based not on dollar values but on the psychological values of outcomes, their utilities. The psychological value of a gamble is therefore not the weighted average of its possible dollar outcomes; it is the average of the utilities of these outcomes, each weighted by its probability.

Table 3 shows a version of the utility function that Bernoulli calculated; it presents the utility of different levels of wealth, from 1 million to 10 million. You can see that adding 1 million to a wealth of 1 million yields an increment of 20 utility points, but adding 1 million to a wealth of 9 million adds only 4 points. Bernoulli proposed that the diminishing marginal value of wealth (in the modern jargon) is what explains risk aversion—the common preference that people generally show for a sure thing over a favorable gamble of equal or slightly higher expected value.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Wealth (millions) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Utility Units | 10 | 30 | 48 | 60 | 70 | 78 | 84 | 90 | 96 | 100 |

Consider this choice:

|  |  |
| --- | --- |
| Equal chances to have 1 million or 7 millions | Utility: (10+84)/2 = 47 |

OR

|  |  |
| --- | --- |
| Have 4 million with certainty | Utility:60 |

The expected value of the gamble and the “sure thing” are equal in ducats (4 million), but the psychological utilities of the two options are different, because of the diminishing utility of wealth: the increment of utility from 1 million to 4 million is 50 units, but an equal increment, from 4 to 7 million, increases the utility of wealth by only 24 units. The utility of the gamble is 94/2 = 47 (the utility of its two outcomes, each weighted by its probability of 1/2). The utility of 4 million is 60. Because 60 is more than 47, an individual with this utility function will prefer the sure thing. Bernoulli’s insight was that a decision maker with diminishing marginal utility for wealth will be risk averse.

Bernoulli’s essay is a marvel of concise brilliance. He applied his new concept of expected utility (which he called “moral expectation”) to compute how much a merchant in St. Petersburg would be willing to pay to insure a shipment of spice from Amsterdam if “he is well aware of the fact that at this time of year of one hundred ships which sail from Amsterdam to Petersburg, five are usually lost.” His utility function explained why poor people buy insurance and why richer people sell it to them. As you can see in the table, the loss of 1 million causes a loss of 4 points of utility (from 100 to 96) to someone who has 10 million and a much larger loss of 18 points (from 48 to 30) to someone who starts off with 3 million. The poorer man will happily pay a premium to transfer the risk to the richer one, which is what insurance is about (eu: lembrar também que a empresa de Seguro ainda trabalha para construir um “Pearson correlation coefficient” negativo entre eventos futuros). Bernoulli also offered a solution to the famous “St. Petersburg paradox,” in which people who are offered a gamble that has infinite expected value (in ducats) are willing to spend only a few ducats for it. Most impressive, his analysis of risk attitudes in terms of preferences for wealth has stood the test of time: it is still current in economic analysis almost 300 years later.

The longevity of the theory is all the more remarkable because it is seriously flawed. The errors of a theory are rarely found in what it asserts explicitly; they hide in what it ignores or tacitly assumes. For an example, take the following scenarios:

Today Jack and Jill each have a wealth of 5 million.

Yesterday, Jack had 1 million and Jill had 9 million.

Are they equally happy? (Do they have the same utility?)

Bernoulli’s theory assumes that the utility of their wealth is what makes people more or less happy. Jack and Jill have the same wealth, and the theory therefore asserts that they should be equally happy, but you do not need a degree in psychology to know that today Jack is elated and Jill despondent. Indeed, we know that Jack would be a great deal happier than Jill even if he had only 2 million today while she has 5. So Bernoulli’s theory must be wrong.

The happiness that Jack and Jill experience is determined by the recent *change* in their wealth, relative to the different states of wealth that define their reference points (1 million for Jack, 9 million for Jill). This reference dependence is ubiquitous in sensation and perception. The same sound will be experienced as very loud or quite faint, depending on whether it was preceded by a whisper or by a roar. To predict the subjective experience of loudness, it is not enough to know its absolute energy; you also need to know the reference sound to which it is automatically compared. Similarly, you need to know about the background before you can predict whether a gray patch on a page will appear dark or light. And you need to know the reference before you can predict the utility of an amount of wealth.

**Chapter 26 - Prospect Theory**

Consider these two problems:

Problem 1: Which do you choose?

Get $900 for sure OR 90% chance to get $1,000

Problem 2: Which do you choose?

Lose $900 for sure OR 90% chance to lose $1,000

You were probably risk averse in problem 1, as is the great majority of people. The subjective value of a gain of $900 is certainly more than 90% of the value of a ga Blth"it ue of a gin of $1,000. The risk-averse choice in this problem would not have surprised Bernoulli.

Now examine your preference in problem 2. If you are like most other people, you chose the gamble in this question. The explanation for this risk-seeking choice is the mirror image of the explanation of risk aversion in problem 1: the (negative) value of losing $900 is much more than 90% of the (negative) value of losing $1,000. The sure loss is very aversive, and this drives you to take the risk. Later, we will see that the evaluations of the probabilities (90% versus 100%) also contributes to both risk aversion in problem 1 and the preference for the gamble in problem 2.

… His theory (Bernoulli’s model) is too simple and lacks a moving part. The missing variable is the reference point, the earlier state relative to which gains and losses are evaluated. In Bernoulli’s theory you need to know only the state of wealth to determine its utility, but in prospect theory you also need to know the reference state. Prospect theory is therefore more complex than utility theory. In science complexity is considered a cost, which must be justified by a sufficiently rich set of new and (preferably) interesting predictions of facts that the existing theory cannot explain. This was the challenge we had to meet.

Although Amos and I were not working with the two-systems model of the mind, it’s clear now that there are three cognitive features at the heart of prospect theory. They play an essential role in the evaluation of financial outcomes and are common to many automatic processes of perception, judgment, and emotion. They should be seen as operating characteristics of System 1.

**a)** Evaluation is relative to a neutral reference point, which is sometimes referred to as an “adaptation level.” You can easily set up a compelling demonstration of this principle. Place three bowls of water in front of you. Put ice water into the left-hand bowl and warm water into the right-hand bowl. The water in the middle bowl should be at room temperature. Immerse your hands in the cold and warm water for about a minute, then dip both in the middle bowl. You will experience the same temperature as heat in one hand and cold in the other. For financial outcomes, the usual reference point is the status quo, but it can also be the outcome that you expect, or perhaps the outcome to which you feel entitled, for example, the raise or bonus that your colleagues receive. Outcomes that are better than the reference points are gains. Below the reference point they are losses.

**b)** A principle of diminishing sensitivity applies to both sensory dimensions and the evaluation of changes of wealth. Turning on a weak light has a large effect in a dark room. The same increment of light may be undetectable in a brightly illuminated room. Similarly, the subjective difference between $900 and $1,000 is much smaller than the difference between $100 and $200.

**c)** The third principle is loss aversion. When directly compared or weighted against each other, losses loom larger than gains. This asymmetry between the power of positive and negative expectations or experiences has an evolutionary history. Organisms that treat threats as more urgent than opportunities have a better chance to survive and reproduce.

The three principles that govern the value of outcomes are illustrated by figure 10. If prospect theory had a flag, this image would be drawn on it. The graph shows the psychological value of gains and losses, which are the “carriers” of value in prospect theory (unlike Bernoulli’s model, in which states of wealth are the carriers of value).

A diagram of a profit

Description automatically generated

The graph has two distinct parts, to the right and to the left of a neutral reference point. A salient feature is that it is ***S-shaped***, which represents diminishing sensitivity for both gains and losses. Finally, the two curves of the ***S*** are not symmetrical. The slope of the function changes abruptly at the reference point: the response to losses is stronger than the response to corresponding gains. **This is loss aversion**.

You can measure the extent of your aversion to losses by asking yourself a question: What is the smallest gain that I need to balance an equal chance to lose $100? For many people the answer is about $200, twice as much as the loss. The “loss aversion ratio” has been estimated in several experiments and is usually in the range of 1.5 to 2.5. This is an average, of course; some people are much more loss averse than others. Professional risk takers in the financial markets are more tolerant of losses, probably because they do not respond emotionally to every fluctuation. When participants in an experiment were instructed to “think like a trader,” they became less loss averse and their emotional reaction to losses (measured by a physiological index of emotional arousal) was sharply reduced (eu: faltou a referência desse experimento).

In order to examine your loss aversion ratio for different stakes, consider the following questions. Ignore any social considerations, do not try to appear either bold or cautious, and focus only on the subjective impact of the possible loss and the offsetting gain.

- Consider a 5 0–5 0 gamble in which you can lose $10. What is the smallest gain that makes the gamble attractive? If you say $10, then you are indifferent to risk. If you give a number less than $10, you seek risk. If your answer is above $10, you are loss averse.

- What about a possible loss of $500 on a coin toss? What possible gain do you require to off set it?

- What about a loss of $2,000?

As you carried out this exercise, you probably found that your loss aversion coefficient tends to increase when the stakes rise, but not dramatically. All bets are off, of course, if the possible loss is potentially ruinous, or if your lifestyle is threatened. The loss aversion coefficient is very large in such cases and may even be infinite - **there are risks that you will not accept, regardless of how many millions you might stand to win if you are lucky.**

**Blind Spots of Prospect Theory**

I emphasized theory-induced blindness in my discussion of flaws in Bernoulli’s model that remained unquestioned for more than two centuries. But of course theory-induced blindness is not restricted to expected utility theory. Prospect theory has flaws of its own *[…]*

In simple words, prospect theory cannot deal with disappointment. Disappointment and the anticipation of disappointment are real, however, and the failure to acknowledge them is as obvious a flow as the counterexamples that I invoked to criticize Bernoulli’s theory.

Prospect theory and utility theory also fail to allow for regret.

**Chapter 29 - The Fourfold Pattern**

One reason for the popularity of the gambling metaphor in the study of decision making is that it provides a natural rule for the assignment of weights to the outcomes of a prospect: the more probable an outcome, the more weight it should have. The expected value of a gamble is the average of its outcomes, each weighted by its probability. For example, the expected value of “20% chance to win $1,000 and 75% chance to win $100” is $275. In the pre-Bernoulli days, gambles were assessed by their expected value. Bernoulli retained this method for assigning weights to the outcomes, which is known as the expectation principle, but applied it to the psychological value of the outcomes. The utility of a gamble, in his theory, is the average of the utilities of its outcomes, each weighted by its probability.

People who buy lottery tickets in vast amounts show themselves willing to pay much more than expected value for very small chances to win a large prize.

The improvement from 95% to 100% is another qualitative change that has a large impact, ***the certainty effect***. Outcomes that are almost certain are given less weight than their probability justifies. To appreciate the certainty effect, imagine that you inherited $1 million, but your greedy stepsister has contested the will in court. The decision is expected tomorrow. Your lawyer assures you that you have a strong case and that you have a 95% chance to win, but he takes pains to remind you that judicial decisions are never perfectly predictable. Now you are approached by a risk-adjustment company, which offers to buy your case for $910,000 outright—take it or leave it. The offer is lower (by $40,000!) than the expected value of waiting for the judgment (which is $950,000), but are you quite sure you would want to reject it? If such an event actually happens in your life, you should know that a large industry of “structured settlements” exists to provide certainty at a hefty price, by taking advantage of the certainty effect.

Possibility and certainty have similarly powerful effects in the domain of losses. When a loved one is wheeled into surgery, a 5% risk that an amputation will be necessary is very bad—much more than half as bad as a 10% risk. Because of the possibility effect, we tend to overweight small risks and are willing to pay far more than expected value to eliminate them altogether. The psychological difference between a 95% risk of disaster and the certainty of disaster appears to be even greater; the sliver of hope that everything could still be okay looms very large. Overweighting of small probabilities increases the attractiveness of both gambles and insurance policies.

**The conclusion is straightforward: the decision weights that people assign to outcomes are not identical to the probabilities of these outcomes**, contrary to the expectation principle. Improbable outcomes are overweighted—this is the possibility effect. Outcomes that are almost certain are underweighted relative to actual certainty. *The expectation principle*, by which values are weighted by their probability, is poor psychology.

**Allais’s Paradox**

*[…]* We went in another direction. We retained utility theory as a logic of rational choice but abandoned the idea that people are perfectly rational choosers. We took on the task of developing a psychological theory that would describe the choices people make, **regardless of whether they are rational**. In prospect theory, **decision weights** would not be identical to probabilities.

Eu: Aqui fica a dúvida: O axioma de racionalidade de von Neuman and Mogerstern está errado??? Ou A Teoria do Prospecto é somente um modelo matemático para calcular como os seres humanos irracionais tomam decisões??? Na minha opinião o axioma de racionalidade de Von Neumann está errado.

**The Fourfold Pattern**

**When Amos and I began our work on prospect theory, we quickly reached two conclusions: people attach values to gains and losses rather than to wealth, and the decision weights that they assign to outcomes are different from probabilities**. Neither idea was completely new, but in combination they explained a distinctive pattern of preferences that we ca Bima ae ca Bimlled the fourfold pattern. The name has stuck. The scenarios are illustrated below.

A white rectangular box with black text

Description automatically generated with medium confidence

The *fourfold pattern* of preferences is considered one of the core achievements of prospect theory.

**Many unfortunate human situations unfold in the top right cell. This is where people who face very bad options take desperate gambles, accepting a high probability of making things worse in exchange for a small hope of avoiding a large loss. Risk taking of this kind often turns manageable failures into disasters. The thought of accepting the large sure loss is too painful, and the hope of complete relief too enticing, to make the sensible decision that it is time to cut one’s losses. This is where businesses that are losing ground to a superior technology waste their remaining assets in futile attempts to catch up. Because defeat is so difficult to accept, the losing side in wars often fights long past the point at which the victory of the other side is certain, and only a matter of time.**

**The decisions described by the fourfold pattern are not obviously unreasonable.** (eu: aqui fica a dúvida: a) a decisão descrita pelo Fourfold pattern é irracional de forma não óbvia???; ou b) ela é não irracional de uma forma que é bastante óbvia de se ver???). You can empathize in each case with the feelings of the plaintiff and the defendant that lead them to adopt a combative or an accommodating posture. In the long run, however, deviations from expected value are likely to be costly. **Consider a large organization, the City of New York, and suppose it faces 200 “frivolous” suits each year, each with a 5% chance to cost the city $1 million. Suppose further that in each case the city could settle the lawsuit for a payment of $100,000.** The city considers two alternative policies that it will apply to all such cases: settle or go to trial. (For simplicity, I ignore legal costs.)

**o** - If the city litigates all 200 cases, it will lose 10, for a total loss of $10million.

**o** - If the city settles every case for $100,000, its total loss will be $20 million.

**When you take the long view of many similar decisions, you can see that paying a premium to avoid a small risk of a large loss is costly.** A similar analysis applies to each of the cells of the fourfold pattern: systematic deviations from expected value are costly in the long run—and this rule applies to both risk aversion and risk seeking. Consistent overweighting of improbable outcomes—a feature of **intuitive decision making**—eventually leads to inferior outcomes.

Eu: Aqui fica a dúvida: O axioma de racionalidade de von Neuman and Mogerstern está errado?? Ou A Teoria do Prospecto é somente um modelo matemático para calcular como “intuitive decision making” é realizado ??? Qual a minha opinião agora???

**Chapter 31 - Risk Policies**

Imagine that you face the following pair of concurrent decisions. First examine both decisions, then make your choices.

Decision (i): Choose between

A. sure gain of $240

B. 25% chance to gain $1,000 and 75% chance to gain nothing

Decision (ii): Choose between

C. sure loss of $750

D. 75% chance to lose $1,000 and 25% chance to lose nothing

This pair of choice problems has an important place in the history of prospect theory, and it has new things to tell us about rationality.

There were tw Bght hecome oo ways of construing decisions i and ii:

**o** - narrow framing: a sequence of two simple decisions, considered separately

**o** - broad framing: a single comprehensive decision, with four options

Broad framing was obviously superior in this case. Indeed, it will be superior (or at least not inferior) in every case in which several decisions are to be contemplated together. Imagine a longer list of 5 simple (binary) decisions to be considered simultaneously. The broad (comprehensive) frame consists of a single choice with 32 options. Narrow framing will yield a sequence of 5 simple choices. The sequence of 5 choices will be one of the 32 options of the broad frame. Will it be the best? Perhaps, but not very likely. **A rational agent will of course engage in broad framing, but Humans are by nature narrow framers.**

**The combination of loss aversion and narrow framing is a costly curse.** Individual investors can avoid that curse, achieving the emotional benefits of broad framing while also saving time and agony, by reducing the frequency with which they check how well their investments are doing. Closely following daily fluctuations is a losing proposition, because the pain of the frequent small losses exceeds the pleasure of the equally frequent small gains. Once a quarter is enough, and may be more than enough for individual investors. In addition to improving the emotional quality of life, the deliberate avoidance of exposure to short-term outcomes improves the quality of both decisions and outcomes. The typical short-term reaction to bad news is increased loss aversion. Investors who get aggregated feedback receive such news much less often and are likely to be less risk averse and to end up richer. You are also less prone to useless churning of your portfolio if you don’t know how every stock in it is doing every day (or every week or even every month). A commitment not to change one’s position for several periods (the equivalent of “locking in” an investment) improves financial performance.

**Risk Policies**

Decision makers who are prone to narrow framing construct a preference every time they face a risky choice. **They would do better by having a risk policy that they routinely apply whenever a relevant problem arises**. Familiar examples of risk policies are “always take the highest possible deductible when purchasing insurance” and “never buy extended warranties.” **A risk policy is a broad frame.** In the insurance examples, you expect the occasional loss of the entire deductible, or the occasional failure of an uninsured product. The relevant issue is your ability to reduce or eliminate the pain of the occasional loss by the thought that the policy that left you exposed to it will almost certainly be financially advantageous over the long run.

**Chapter 32 - Keeping Score**

The escalation of commitment to failing endeavors is a mistake from the perspective of the firm but not necessarily from the perspective of the executive who “owns” a floundering project. Canceling the project will leave a permanent stain on the executive’s record, and his personal interests are perhaps best served by gambling further with the organization’s resources in the hope of recouping the original investment—or at least in an attempt to postpone the day of reckoning. **In the presence of sunk costs, the manager’s incentives are misaligned with the objectives of the firm and its shareholders, a familiar type of what is known as the agency problem.** Boards of directors are well aware of these conflicts and often replace a CEO who is encumbered by prior decisions and reluctant to cut losses. The members of the board do not necessarily believe that the new CEO is more competent than the one she replaces. They do know that she does not carry the same mental accounts and is therefore better able to ignore the sunk costs of past investments in evaluating current opportunities.

**The sunk-cost fallacy** keeps people for too long in poor jobs, unhappy marriages, and unpromising research projects. I have often observed young scientists struggling to salvage a doomed project when they would be better advised to drop it and start a new one. Fortunately, research suggests that at least in some contexts the fallacy can be overcome. **The sunk-cost fallacy is identified and taught as a mistake in both economics and business courses, apparently to good effect: there is evidence that graduate students in these fields are more willing than others to walk away from a failing project. (eu: remember The Concorde fallacy)**